

true or false, though it is equally a fact in either case. One *might* call wishes false in the same sense when one wishes something that does not happen. The truth or falsehood depends upon the proposition that enters in. I am inclined to think that perception, as opposed to belief, does go straight to the fact and not through the proposition. When you perceive the fact you do not, of course, have error coming in, because the moment it is a fact that is your object error is excluded. I think that verification in the last resort would always reduce itself to the perception of facts. Therefore the logical form of perception will be different from the logical form of believing, just because of that circumstance that it is a *fact* that comes in. That raises also a number of logical difficulties which I do not propose to go into, but I think you can see for yourself that perceiving would also involve two verbs just as believing does. I am inclined to think that volition differs from desire logically, in a way strictly analogous to that in which perception differs from belief. But it would take us too far from logic to discuss this view.

V. GENERAL PROPOSITIONS AND EXISTENCE

I am going to speak to-day about general propositions and existence. The two subjects really belong together; they are the same topic, although it might not have seemed so at the first glance. The propositions and facts that I have been talking about hitherto have all been such as involved only perfectly definite particulars, or relations, or qualities, or things of that sort, never involved the sort of indefinite things one alludes to by such words as 'all', 'some', 'a', 'any', and it is propositions and facts of that sort that I am coming on to to-day.

Really all the propositions of the sort that I mean to talk of to-day collect themselves into two groups—the *first* that are about 'all', and the *second* that are about 'some'. These two sorts belong together; they are each other's negations. If you say, for instance, 'All men are mortal', that is the negative of 'Some men are not mortal'. In regard to general propositions, the distinction of affirmative and negative is arbitrary. Whether you are going to regard the propositions about 'all' as the affirmative ones and the propositions about 'some' as the negative ones, or vice versa, is purely a

matter of taste. For example, if I say 'I met no one as I came along', that, on the face of it, you would think is a negative proposition. Of course, that is really a proposition about 'all', i.e., 'All men are among those whom I did not meet'. If, on the other hand, I say 'I met a man as I came along', that would strike you as affirmative, whereas it is the negative of 'All men are among those I did not meet as I came along'. If you consider such propositions as 'All men are mortal' and 'Some men are not mortal', you might say it was more natural to take the general propositions as the affirmative and the existence-propositions as the negative, but, simply because it is quite arbitrary which one is to choose, it is better to forget these words and to speak only of general propositions and propositions asserting existence. All general propositions deny the existence of something or other. If you say 'All men are mortal', that denies the existence of an immortal man, and so on.

I want to say emphatically that general propositions are to be interpreted as not involving existence. When I say, for instance, 'All Greeks are men', I do not want you to suppose that that implies that there are Greeks. It is to be considered emphatically as not implying that. That would have to be added as a separate proposition. If you want to interpret it in that sense, you will have to add the further statement 'and there are Greeks'. That is for purposes of practical convenience. If you include the fact that there are Greeks, you are rolling two propositions into one, and it causes unnecessary confusion in your logic, because the sorts of propositions that you want are those that do assert the existence of something and general propositions which do not assert existence. If it happened that there were no Greeks, both the proposition that 'All Greeks are men' and the proposition that 'No Greeks are men' would be true. The proposition 'No Greeks are men' is, of course, the proposition 'All Greeks are not-men'. Both propositions will be true simultaneously if it happens that there are no Greeks. All statements about all the members of a class that has no members are true, because the contradictory of any general statement does assert existence and is therefore false in this case. This notion, of course, of general propositions not involving existence is one which is not in the traditional doctrine of the syllogism. In the traditional doctrine of the syllogism, it was assumed that when you have such a statement as 'All Greeks are men', that implies

that there are Greeks, and this produced fallacies. For instance, 'All chimeras are animals, and all chimeras breathe flame, therefore some animals breathe flame.' This is a syllogism in Darapti, but that mood of the syllogism is fallacious, as this instance shows. That was a point, by the way, which had a certain historical interest, because it impeded Leibniz in his attempts to construct a mathematical logic. He was always engaged in trying to construct such a mathematical logic as we have now, or rather such a one as Boole constructed, and he was always failing because of his respect for Aristotle. Whenever he invented a really good system, as he did several times, it always brought out that such moods as Darapti are fallacious. If you say 'All A is B and all A is C , therefore some B is C '—if you say this you incur a fallacy, but he could not bring himself to believe that it was fallacious, so he began again. That shows you that you should not have too much respect for distinguished men.*

Now when you come to ask what really is asserted in a general proposition, such as 'All Greeks are men' for instance, you find that what is asserted is the truth of all values of what I call a propositional function. A *propositional function* is simply any expression containing an undetermined constituent, or several undetermined constituents, and becoming a proposition as soon as the undetermined constituents are determined. If I say ' x is a man' or ' n is a number', that is a propositional function; so is any formula of algebra, say $(x+y)(x-y) = x^2 - y^2$. A propositional function is nothing, but, like most of the things one wants to talk about in logic, it does not lose its importance through that fact. The only thing really that you can do with a propositional function is to assert either that it is always true, or that it is sometimes true, or that it is never true. If you take:

'If x is a man, x is mortal',

that is always true (just as much when x is not a man as when x is a man); if you take:

' x is a man',

that is sometimes true; if you take:

' x is a unicorn',

that is never true.

* Cf. Couturat, *La logique de Leibniz*.

One may call a propositional function

necessary, when it is always true;
possible, when it is sometimes true;
impossible, when it is never true.

Much false philosophy has arisen out of confusing propositional functions and propositions. There is a great deal in ordinary traditional philosophy which consists simply in attributing to propositions the predicates which only apply to propositional functions, and, still worse, sometimes in attributing to individuals predicates which merely apply to propositional functions. This case of *necessary*, *possible*, *impossible*, is a case in point. In all traditional philosophy there comes a heading of 'modality', which discusses *necessary*, *possible*, and *impossible* as properties of propositions, whereas in fact they are properties of propositional functions. Propositions are only true or false.

If you take ' x is x ', that is a propositional function which is true whatever ' x ' may be, i.e., a necessary propositional function. If you take ' x is a man', that is a possible one. If you take ' x is a unicorn', that is an impossible one.

Propositions can only be true or false, but propositional functions have these three possibilities. It is important, I think, to realize that the whole doctrine of modality only applies to propositional functions, not to propositions.

Propositional functions are involved in ordinary language in a great many cases where one does not usually realize them. In such a statement as 'I met a man', you can understand my statement perfectly well without knowing whom I met, and the actual person is not a constituent of the proposition. You are really asserting there that a certain propositional function is sometimes true, namely the propositional function 'I met x and x is human'. There is at least one value of x for which that is true, and that therefore is a possible propositional function. Whenever you get such words as 'a', 'some', 'all', 'every', it is always a mark of the presence of a propositional function, so that these things are not, so to speak, remote or recondite: they are obvious and familiar.

A propositional function comes in again in such a statement as 'Socrates is mortal', because 'to be mortal' means 'to die at some time or other'. You mean there is a time at which Socrates dies,

and that again involves a propositional function, namely, that ' t is a time, and Socrates dies at t ' is possible. If you say 'Socrates is immortal', that also will involve a propositional function. That means that 'If t is any time whatever, Socrates is alive at time t ', if we take immortality as involving existence throughout the whole of the past as well as throughout the whole of the future. But if we take immortality as only involving existence throughout the whole of the future, the interpretation of 'Socrates is immortal' becomes more complete, viz., 'There is a time t , such that if t' is any time later than t , Socrates is alive at t' .' Thus when you come to write out properly what one means by a great many ordinary statements, it turns out a little complicated. 'Socrates is mortal' and 'Socrates is immortal' are not each other's contradictories, because they both imply that Socrates exists in time, otherwise he would not be either mortal or immortal. One says, 'There is a time at which he dies', and the other says, 'Whatever time you take, he is alive at that time', whereas the contradictory of 'Socrates is mortal' would be true if there is not a time at which he lives.

An undetermined constituent in a propositional function is called a *variable*.

Existence. When you take any propositional function and assert of it that it is possible, that it is sometimes true, that gives you the fundamental meaning of 'existence'. You may express it by saying that there is at least one value of x for which that propositional function is true. Take ' x is a man', there is at least one value of x for which this is true. That is what one means by saying that 'There are men', or that 'Men exist'. Existence is essentially a property of a propositional function. It means that that propositional function is true in at least one instance. If you say 'There are unicorns', that will mean that 'There is an x , such that x is a unicorn'. That is written in phrasing which is unduly approximated to ordinary language, but the proper way to put it would be '(x is a unicorn) is possible'. We have got to have some idea that we do not define, and one takes the idea of 'always true', or of 'sometimes true', as one's undefined idea in this matter, and then you can define the other one as the negative of that. In some ways it is better to take them both as undefined, for reasons which I shall not go into at present. It will be out of this notion of *sometimes*, which is the same as the notion of *possible*, that we get the notion

of existence. To say that unicorns exist is simply to say that '(x is a unicorn) is possible'.

It is perfectly clear that when you say 'Unicorns exist', you are not saying anything that would apply to any unicorns there might happen to be, because as a matter of fact there are not any, and therefore if what you say had any application to the actual individuals, it could not possibly be significant unless it were true. You can consider the proposition 'Unicorns exist' and can see that it is false. It is not nonsense. Of course, if the proposition went through the general conception of the unicorn to the individual, it could not be even significant unless there were unicorns. Therefore when you say 'Unicorns exist', you are not saying anything about any individual things, and the same applies when you say 'Men exist'. If you say that 'Men exist, and Socrates is a man, therefore Socrates exists', that is exactly the same sort of fallacy as it would be if you said 'Men are numerous, Socrates is a man, therefore Socrates is numerous', because existence is a predicate of a propositional function, or derivatively of a class. When you say of a propositional function that it is numerous, you will mean that there are several values of x that will satisfy it, that there are more than one; or, if you like to take 'numerous' in a larger sense, more than ten, more than twenty, or whatever number you think fitting. If x , y , and z all satisfy a propositional function, you may say that that proposition is numerous, but x , y , and z severally are not numerous. Exactly the same applies to existence, that is to say that the actual things that there are in the world do not exist, or, at least, that is putting it too strongly, because that is utter nonsense. To say that they do not exist is strictly nonsense, but to say that they do exist is also strictly nonsense.

It is of propositional functions that you can assert or deny existence. You must not run away with the idea that this entails consequences that it does not entail. If I say 'The things that there are in the world exist', that is a perfectly correct statement, because I am there saying something about a certain class of things; I say it in the same sense in which I say 'Men exist'. But I must not go on to 'This is a thing in the world, and therefore this exists'. It is there the fallacy comes in, and it is simply, as you see, a fallacy of transferring to the individual that satisfies a propositional function a predicate which only applies to a propositional function.

You can see this in various ways. For instance, you sometimes know the truth of an existence-proposition without knowing any instance of it. You know that there are people in Timbuctoo, but I doubt if any of you could give me an instance of one. Therefore you clearly can know existence-propositions without knowing any individual that makes them true. Existence-propositions do not say anything about the actual individual but only about the class or function.

It is exceedingly difficult to make this point clear as long as one adheres to ordinary language, because ordinary language is rooted in a certain feeling about logic, a certain feeling that our primeval ancestors had, and as long as you keep to ordinary language you find it very difficult to get away from the bias which is imposed upon you by language. When I say, e.g., 'There is a x such that x is a man', that is not the sort of phrase one would like to use. 'There is an x ' is meaningless. What is 'an x ' anyhow? There is not such a thing. The only way you can really state it correctly is by inventing a new language *ad hoc*, and making the statement apply straight off to ' x is a man', as when one says '(x is a man) is possible', or invent a special symbol for the statement that ' x is a man' is sometimes true.

I have dwelt on this point because it really is of very fundamental importance. I shall come back to existence in my next lecture: existence as it applies to descriptions, which is a slightly more complicated case than I am discussing here. I think an almost unbelievable amount of false philosophy has arisen through not realizing what 'existence' means.

As I was saying a moment ago, a propositional function in itself is nothing: it is merely a schema. Therefore in the inventory of the world, which is what I am trying to get at, one comes to the question: What is there really in the world that corresponds with these things? Of course, it is clear that we have general *propositions*, in the same sense in which we have atomic propositions. For the moment I will include existence-propositions with general propositions. We have such propositions as 'All men are mortal' and 'Some men are Greeks'. But you have not only such *propositions*; you have also such *facts*, and that, of course, is where you get back to the inventory of the world: that, in addition to particular facts, which I have been talking about in previous lectures, there

are also general facts and existence-facts, that is to say, there are not merely *propositions* of that sort but also *facts* of that sort. That is rather an important point to realize. You cannot ever arrive at a general fact by inference from particular facts, however numerous. The old plan of complete induction, which used to occur in books, which was always supposed to be quite safe and easy as opposed to ordinary induction, that plan of complete induction, unless it is accompanied by at least one general proposition, will not yield you the result that you want. Suppose, for example, that you wish to prove in that way that 'All men are mortal', you are supposed to proceed by complete induction, and say '*A* is a man that is mortal', '*B* is a man that is mortal', '*C* is a man that is mortal', and so on until you finish. You will not be able, in that way, to arrive at the proposition 'All men are mortal' unless you know when you have finished. That is to say that, in order to arrive by this road at the general proposition 'All men are mortal', you must already have the general proposition 'All men are among those I have enumerated'. You never can arrive at a general proposition by inference from particular propositions alone. You will always have to have at least one general proposition in your premises. That illustrates, I think, various points. One, which is epistemological, is that if there is, as there seems to be, knowledge of general propositions, then there must be *primitive* knowledge of general propositions (I mean by that, knowledge of general propositions which is not obtained by inference), because if you can never infer a general proposition except from premises of which one at least is general, it is clear that you can never have knowledge of such propositions by inference unless there is knowledge of some general propositions which is not by inference. I think that the sort of way such knowledge—or rather the belief that we have such knowledge—comes into ordinary life is probably very odd. I mean to say that we do habitually assume general propositions which are exceedingly doubtful; as, for instance, one might, if one were counting up the people in this room, assume that one could see all of them, which is a general proposition, and very doubtful as there may be people under the tables. But, apart from that sort of thing, you do have in any empirical verification of general propositions some kind of assumption that amounts to this, that what you do not see is not there. Of course, you would not put it so

strongly as that, but you would assume that, with certain limitations and certain qualifications, if a thing does not appear to your senses, it is not there. That is a general proposition, and it is only through such propositions that you arrive at the ordinary empirical results that one obtains in ordinary ways. If you take a census of the country, for instance, you assume that the people you do not see are not there, provided you search properly and carefully, otherwise your census might be wrong. It is some assumption of that sort which would underlie what seems purely empirical. You could not prove empirically that what you do not perceive is not there, because an empirical proof would consist in perceiving, and by hypothesis you do not perceive it, so that any proposition of that sort, if it is accepted, has to be accepted on its own evidence. I only take that as an illustration. There are many other illustrations one could take of the sort of propositions that are commonly assumed, many of them with very little justification.

I come now to a question which concerns logic more nearly, namely, the reasons for supposing that there are general facts as well as general propositions. When we were discussing molecular propositions I threw doubt upon the supposition that there are molecular facts, but I do not think one can doubt that there are general facts. It is perfectly clear, I think, that when you have enumerated all the atomic facts in the world, it is a further fact about the world that those are all the atomic facts there are about the world, and that is just as much an objective fact about the world as any of them are. It is clear, I think, that you must admit general facts as distinct from and over and above particular facts. The same thing applies to 'All men are mortal'. When you have taken all the particular men that there are, and found each one of them severally to be mortal, it is definitely a new fact that all men are mortal; how new a fact, appears from what I said a moment ago, that it could not be inferred from the mortality of the several men that there are in the world. Of course, it is not so difficult to admit what I might call existence-facts—such facts as 'There are men', 'There are sheep', and so on. Those, I think, you will readily admit as separate and distinct facts over and above the atomic facts I spoke of before. Those facts have got to come into the inventory of the world, and in that way propositional functions come in as involved in the study of general facts. I do not profess

to know what the right analysis of general facts is. It is an exceedingly difficult question, and one which I should very much like to see studied. I am sure that, although the convenient technical treatment is by means of propositional functions, that is not the whole of the right analysis. Beyond that I cannot go.

There is one point about whether there are molecular facts. I think I mentioned, when I was saying that I did not think there were disjunctive facts, that a certain difficulty does arise in regard to general facts. Take 'All men are mortal'. That means:

"x is a man" implies
"x is a mortal" whatever
x may be.'

You can see at once that it is a hypothetical proposition. It does not imply that there are any men, nor who are men, and who are not; it simply says that if you have anything which is a man, that thing is mortal. As Mr. Bradley has pointed out in the second chapter of his *Principles of Logic*, 'Trespassers will be prosecuted' may be true even if no one trespasses, since it means merely that, *if* any one trespasses, he will be prosecuted. It comes down to this that

"x is a man" implies "x is a mortal" is always true',

is a fact. It is perhaps a little difficult to see how that can be true if one is going to say that "'Socrates is a man" implies "Socrates is a mortal"' is not itself a fact, which is what I suggested when I was discussing disjunctive facts. I do not feel sure that you could not get round that difficulty. I only suggest it as a point which should be considered when one is denying that there are molecular facts, since, if it cannot be got round, we shall have to admit molecular facts.

Now I want to come to the subject of *completely general* propositions and propositional functions. By those I mean propositions and propositional functions that contain only variables and nothing else at all. This covers the whole of logic. Every logical proposition consists wholly and solely of variables, though it is not true that every proposition consisting wholly and solely of variables is logical. You can consider stages of generalizations as, e.g.,

'Socrates loves Plato'
 ' x loves Plato'
 ' x loves y '
 ' $x R y$ '

There you have been going through a process of successive generalization. When you have got to xRy , you have got a schema consisting only of variables, containing no constants at all, the pure schema of dual relations, and it is clear that any proposition which expresses a dual relation can be derived from xRy by assigning values to x and R and y . So that that is, as you might say, the pure form of all those propositions. I mean by the form of a proposition that which you get when for every single one of its constituents you substitute a variable. If you want a different definition of the form of a proposition, you might be inclined to define it as the class of all those propositions that you can obtain from a given one by substituting other constituents for one or more of the constituents the proposition contains. E.g., in 'Socrates loves Plato', you can substitute somebody else for Socrates, somebody else for Plato, and some other verb for 'loves'. In that way there are a certain number of propositions which you can derive from the proposition 'Socrates loves Plato', by replacing the constituents of that proposition by other constituents, so that you have there a certain class of propositions, and those propositions all have a certain form, and one can, if one likes, say that the form they all have is the class consisting of all of them. That is rather a provisional definition, because as a matter of fact, the idea of form is more fundamental than the idea of class. I should not suggest that as a really good definition, but it will do provisionally to explain the sort of thing one means by the form of a proposition. The form of a proposition is that which is in common between any two propositions of which the one can be obtained from the other by substituting other constituents for the original ones. When you have got down to those formulas that contain only variables, like xRy , you are on the way to the sort of thing that you can assert in logic.

To give an illustration, you know what I mean by the domain of a relation: I mean all the terms that have that relation to something. Suppose I say: ' xRy implies that x belongs to the domain of R ',

that would be a proposition of logic and is one that contains only variables. You might think it contains such words as 'belong' and 'domain', but that is an error. It is only the habit of using ordinary language that makes those words appear. They are not really there. That is a proposition of pure logic. It does not mention any particular thing at all. This is to be understood as being asserted whatever x and R and y may be. All the statements of logic are of that sort.

It is not a very easy thing to see what are the constituents of a logical proposition. When one takes 'Socrates loves Plato', 'Socrates' is a constituent, 'loves' is a constituent, and 'Plato' is a constituent. Then you turn 'Socrates' into x , 'loves' into R , and 'Plato' into y . x and R and y are nothing, and they are not constituents, so it seems as though all the propositions of logic were entirely devoid of constituents. I do not think that can quite be true. But then the only other thing you can seem to say is that the *form* is a constituent, that propositions of a certain form are always true: that *may* be the right analysis, though I very much doubt whether it is.

There is, however, just this to observe, viz., that the form of a proposition is never a constituent of that proposition itself. If you assert that 'Socrates loves Plato', the form of that proposition is the form of the dual relation, but this is not a constituent of the proposition. If it were you would have to have that constituent related to the other constituents. You will make the form much too substantial if you think of it as really one of the things that have that form, so that the form of a proposition is certainly not a constituent of the proposition itself. Nevertheless it may possibly be a constituent of general statements about propositions that have that form, so I think it is *possible that* logical propositions might be interpreted as being about forms.

I can only say, in conclusion, as regards the constituents of logical propositions, that it is a problem which is rather new. There has not been much opportunity to consider it. I do not think any literature exists at all which deals with it in any way whatever, and it is an interesting problem.

I just want now to give you a few illustrations of propositions which can be expressed in the language of pure variables but are not propositions of logic. Among the propositions that are propositions of logic are included all the propositions of pure mathematics, all of which cannot only be expressed in logical terms but

can also be deduced from the premises of logic, and therefore they are logical propositions. Apart from them there are many that can be expressed in logical terms, but cannot be proved from logic, and are certainly not propositions that form part of logic. Suppose you take such a proposition as: 'There is at least one thing in the world'. That is a proposition that you can express in logical terms. It will mean, if you like, that the propositional function ' $x = x$ ' is a possible one. That is a proposition, therefore, that you can express in logical terms; but you cannot know from logic whether it is true or false. So far as you do know it, you know it empirically, because there might happen not to be a universe, and then it would not be true. It is merely an accident, so to speak, that there is a universe. The proposition that there are exactly 30,000 things in the world can also be expressed in purely logical terms, and is certainly not a proposition of logic but an empirical proposition (true or false), because a world containing more than 30,000 things and a world containing fewer than 30,000 things are both possible, so that if it happens that there are exactly 30,000 things, that is what one might call an accident and is not a proposition of logic. There are again two propositions that one is used to in mathematical logic, namely, the multiplicative axiom and the axiom of infinity. These also can be expressed in logical terms, but cannot be proved or disproved by logic. In regard to the axiom of infinity, the impossibility of logical proof or disproof may be taken as certain, but in the case of the multiplicative axiom, it is perhaps still open to some degree to doubt. Everything that is a proposition of logic has got to be in some sense or other like a tautology. It has got to be something that has some peculiar quality, which I do not know how to define, that belongs to logical propositions and not to others. Examples of typical logical propositions are:

'If p implies q and q implies r , then p implies r .'

'If all a 's are b 's and all b 's are c 's, then all a 's are c 's.'

'If all a 's are b 's, and x is an a , then x is a b .'

Those are propositions of logic. They have a certain peculiar quality which marks them out from other propositions and enables us to know them *a priori*. But what exactly that characteristic is, I am not able to tell you. Although it is a necessary characteristic of logical propositions that they should consist solely of variables,

i.e., that they should assert the universal truth, or the sometimes-truth, of a propositional function consisting wholly of variables—although that is a necessary characteristic, it is not a sufficient one. I am sorry that I have had to leave so many problems unsolved. I always have to make this apology, but the world really is rather puzzling and I cannot help it.

Discussion

Question: Is there any word you would substitute for 'existence' which would give existence to individuals? Are you applying the word 'existence' to two ideas, or do you deny that there are two ideas?

Mr. Russell: No, there is not an idea that will apply to individuals. As regards the actual things there are in the world, there is nothing at all you can say about them that in any way corresponds to this notion of existence. It is a sheer mistake to say that there is anything analogous to existence that you can say about them. You get into confusion through language, because it is a perfectly correct thing to say 'All the things in the world exist', and it is so easy to pass from this to 'This exists because it is a thing in the world'. There is no sort of point in a predicate which could not conceivably be false. I mean, it is perfectly clear that, if there were such a thing as this existence of individuals that we talk of, it would be absolutely impossible for it not to apply, and that is the characteristic of a mistake.

VI. DESCRIPTIONS AND INCOMPLETE SYMBOLS

I am proposing to deal this time with the subject of descriptions, and what I call 'incomplete symbols', and the existence of described individuals. You will remember that last time I dealt with the existence of *kinds* of things, what you mean by saying 'There are men' or 'There are Greeks' or phrases of that sort, where you have an existence which may be plural. I am going to deal to-day with an existence which is asserted to be singular, such as 'The man with the iron mask existed' or some phrase of that sort, where you have some object described by the phrase 'The so-and-so' in the singular, and I want to discuss the analysis of propositions in which phrases of that kind occur.