An Introduction to LOGIC and SCIENTIFIC METHOD

BY

MORRIS R. COHEN

Department of Philosophy, College of the City of New York

AND

ERNEST NAGEL

Department of Philosophy, Columbia University

HARCOURT, BRACE AND COMPANY

NEW YORK

if a and b are any two propositions $(a \supset b) \equiv (b' \supset a')$. We now have:

$$[(p \cdot q) \supset r] \equiv [(q \cdot r') \supset p'] \equiv [(p \cdot r') \supset q']$$

The principle of indirect reduction may therefore be analyzed as follows: *The syllogism is a form of inference in which two propositions* p *and* q *jointly imply a third* r, *where the three propositions contain three and only three terms.* If, however, we deny the implication $[(p \cdot q) \supset r]$, we must also deny a second implication which is equivalent to it: $[(q \cdot r') \supset p']$. But this second implication in our illustration above was a valid syllogism in *Barbara,* which cannot be denied. Therefore the first implication, which represents an *OAO* syllogism in the third figure (*Bocardo*) cannot be doubted either. For the denial of the validity of *Bocardo* commits us to the denial of the validity of *Barbara,* which is absurd.

If weakened and strengthened forms are not permitted (that is, if we do not assume existential import for universal propositions), reduction enables us to see that all syllogistic arguments can be reduced to two forms: one in which both premises are universal, and the other in which one premise is particular. The former is an argument in which both propositions may be pure hypotheses; the latter involves statements of fact ultimately dependent on observation.

§ 11. THE ANTILOGISM OR INCONSISTENT TRIAD

The principle involved in indirect reduction has been extended by Mrs. Christine Ladd Franklin in such a way as to provide a new and very powerful method for testing the validity of any syllogism. We shall, however, in discussing this method drop the assumption we have made concerning the existence of the classes denoted by the terms of the syllogism. As a consequence, the weakened and strengthened moods must be eliminated as invalid.

Consider the valid syllogism:

All musicians are proud.
All Scotchmen are musicians.
∴ All Scotchmen are proud.

If we let *S, M,* and *P* symbolize the terms "Scotchmen," "musicians," and "proud individuals," and if we make use of the analysis we have given of what asserted is by categorical proposition in

Chapter IV, this syllogism must be interpreted to assert the following:

$$\begin{array}{r} M\bar{P} = 0 \\ S\bar{M} = 0 \\ \therefore\ S\bar{P} = 0 \end{array}$$

Now if the premises *All musicians are proud* and *All Scotchmen are musicians* necessarily imply *All Scotchmen are proud,* it follows that these premises are incompatible with the *contradictory* of this conclusion. Hence the three propositions:

1. All musicians are proud.
2. All Scotchmen are musicians.
3. Some Scotchmen are not proud:

are *inconsistent* with one another. They cannot all three be true together. Symbolically stated,

$$\begin{array}{r} M\bar{P} = 0 \\ S\bar{M} = 0 \\ S\bar{P} \neq 0 \end{array}$$

are inconsistent. A triad of propositions two of which are the premises of a valid syllogism while the third is the contradictory of its conclusion, is called an *antilogism* or *inconsistent triad.*

An examination of the antilogism above reveals, however, that any two propositions of the triad necessarily imply the *contradictory* of the third. (This can be shown to be true in general, and is a further extension of the equivalence between a hypothetical proposition and its contrapositive.) Thus, if we take the first two of the triad as premises, we get:

All musicians are proud.
All Scotchmen are musicians.
∴ All Scotchmen are proud:

$$\begin{array}{r} M\bar{P} = 0 \\ S\bar{M} = 0 \\ \therefore\ S\bar{P} = 0 \end{array}$$

which is the original syllogism from which the triad was obtained. If we take the first and third of the triad as premises, we get:

All musicians are proud.
Some Scotchmen are not proud.
∴ Some Scotchmen are not musicians:

$$\begin{array}{r} M\bar{P} = 0 \\ S\bar{P} \neq 0 \\ \therefore\ S\bar{M} \neq 0 \end{array}$$

which is a valid mood in the second figure. Finally, if we take the second and third of the triad as premises, we get:

All Scotchmen are musicians. Some Scotchmen are not proud. ∴ Some musicians are not proud:	$S\bar{M} = 0$ $S\bar{P} \neq 0$ $\therefore M\bar{P} \neq 0$

which is a valid mood in the third figure.

The reader is advised to take a different valid mood of the syllogism, and obtain from it the inconsistent triad and the other two valid syllogisms to which it is equivalent.

The Structure of the Antilogism

Let us now examine the structure of the antilogism. The reader will note, in the first place, that it contains two universal propositions and one particular proposition. This is the same as saying that in the symbolic representation of the members of the triad, there are two *equations,* and one *inequation,* because a universal proposition is interpreted as denying existence, while a particular proposition asserts it. Confining his attention to the symbolic representation, the reader will find, in the second place, that the two universals have a common term, which is once positive and once negative. Finally, the particular proposition contains the other two terms. It can be shown without difficulty that these three conditions are present in every antilogism, and the reader should not hesitate to prove that this is so.

Now since every valid syllogism corresponds to an antilogism, we can employ the conditions we have discovered in every antilogism as a test for the validity of any syllogism. Hence it is possible to develop the theory of the categorical syllogism on the basis of the conditions for the antilogism. The single principle required is: *A syllogism is valid if it corresponds to an antilogism whose structure conforms to the three conditions above.*

The theory of the antilogism represents an attempt to discover a more general basis for the syllogism and other inferences studied in traditional logic. The reader will note the elegance and the power which result from the introduction of specially designed symbols. We shall indicate in the following chapter the close connection between advances in logical theory and improvement in symbolism. We will conclude this discussion, however, by indicating how the antilogism may be used to test syllogisms for their validity.

Is the following valid?

Some Orientals are polite.
All Orientals are shrewd.
∴ Some shrewd people are polite.

Letting *S, P, O* stand for the minor, major, and middle terms respectively, the symbolic equivalent of this inference is: $OP \neq 0$, $O\bar{S} = 0$, $\therefore SP \neq 0$. The equivalent antilogism is: $OP \neq 0$, $O\bar{S} = 0$, $SP = 0$. This contains two universals and one particular; the universals have a common term which is once positive and once negative; and the particular contains the other two terms. The syllogism is therefore valid.

Is the following valid?

Some professors are not married.
All saints are married.
∴ Some saints are not professors.

Letting *S, P, M* stand for the minor, major, and middle terms, this may be stated symbolically as: $P\bar{M} \neq 0$, $S\bar{M} = 0$, $S\bar{P} \neq 0$. The equivalent antilogism is: $P\bar{M} \neq 0$, $S\bar{M} = 0$, $S\bar{P} = 0$. This contains two universals and one particular, but the common term in the former is not positive once and negative once. Hence the syllogism is invalid.

§ 12. THE SORITES

It sometimes happens that the evidence for a conclusion consists of more than two propositions. The inference is not a syllogism in such cases, and the examination of all possible ways in which more than two propositions may be combined to yield a conclusion requires a more general approach to logic than the traditional discussions make possible—or an elementary treatise permits. In certain special cases, however, the principles of the syllogism enable us to evaluate such more complex inferences. Thus, from the premises:

All dictatorships are undemocratic.
All undemocratic governments are unstable.
All unstable governments are cruel.
All cruel governments are objects of hate:

we may infer the conclusion:

All dictatorships are objects of hate.

The inference may be tested by means of the syllogistic rules, for the argument is a *chain* of syllogisms in which the conclusion of one becomes a premise of another. In this illustration, however, the conclusions of all the syllogisms except the last remain unexpressed. A chain of syllogisms in which the conclusion of one is a premise in another, in which all the conclusions except the last one are unexpressed, and in which the premises are so arranged that any two successive ones contain a common term, is called a *sorites.*

The above illustration is an *Aristotelian sorites.* In it, the first premise contains the subject of the conclusion, and the common term of two successive propositions appears first as a predicate and next as a subject. A second form of sorites is the *Goclenian sorites.* The following illustrates it:

All sacred things are protected by the state.
All property is sacred.
All trade monopolies are property.
All steel industries are trade monopolies.
∴ All steel industries are protected by the state.

Here the first premise contains the predicate of the conclusion, and the common term of two successive propositions appears first as subject and next as predicate.

Special rules for the sorites may be given. We shall state them and leave their proof as an exercise for the reader.

Special Rules for the Aristotelian Sorites.

1. No more than one premise may be negative; if a premise is negative, it must be the last.
2. No more than one premise may be particular; if a premise is particular, it must be the first.

Special Rules for the Goclenian Sorites.

1. No more than one premise may be negative; if a premise is negative, it must be the first.
2. No more than one premise may be particular; if a premise is particular, it must be the last.