

# LOGIC

## PART I

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πάντα ῥεῖ εἰ μὴ τὸ ἀληθές

DOVER PUBLICATIONS, INC.  
NEW YORK

architect, or in the class hypocrite? If this question is answered in the negative it must be on the ground that, in some sense of the term 'exist' which is not appropriate to *classes*, Mr Pecksniff does not and never did exist, and hence he cannot count as one when we are enumerating the members comprised in any given class. Furthermore, since the numerical predication 'at least one' is highly indeterminate and could be in this or that case replaced by the relatively determinate 'at least  $n$ ' where  $n$  stands for this or that number, the affirmation that 'the class  $C$  exists' is only a special and less determinate case of the affirmation that 'the class  $C$  comprises at least  $n$  items,' and the number  $n$  cannot be counted as such unless all the  $n$  items *exist*. The conclusion therefore follows that the sense of the word 'exist' when predicated of a class is dependent upon that of the word 'exist' when predicated of an item or individual indicated by a proper or uniquely descriptive name<sup>1</sup>.

<sup>1</sup> This contention is directed against the position held in the *Principia Mathematica*, where  $E!$  is ultimately defined in terms of  $\exists$ , whereas in my view  $\exists$  is to be ultimately defined in terms of  $E!$

## CHAPTER XI

## THE DETERMINABLE

§ 1. IN this chapter we propose to discuss a certain characteristic of the adjective as such, which perhaps throws the strongest light upon the antithesis between it and the substantive. Here it will be apposite to consider the traditional account of the principles of logical division where a class (of substantives) is represented as consisting of sub-classes. This process is governed by the following rules: (1) the sub-classes must be mutually exclusive; (2) they must be collectively exhaustive of the class to be divided; (3) division of the class into its co-ordinate sub-classes must be based upon some one 'fundamentum divisionis.' The first two of these rules may be said to be purely formal, and do not raise any problem of immediate interest; but the technical term *fundamentum divisionis*—though perhaps readily understood by the learner—is actually introduced without explicit account of its connection with, or its bearing upon, ideas which have entered into the previous logical exposition. To illustrate the notion we are told, for instance, that, when a class of things is to be divided according to colour, or to size, or to some other aspect in which they can be compared, then the colour, size, or other aspect constitutes the *fundamentum divisionis*. Now although, grammatically speaking, words like colour and size are substantival, they are in

fact abstract names which stand for adjectives; so that the fundamentum divisionis is, in the first place, an adjective, and in the second, an adjective of the particular kind illustrated by 'colour' when considered in its relation to red, blue, green, etc. Superficially this relation appears to be the same as that of a single object to some class of which it is a member: thus two such propositions as 'Red is a colour' and 'Plato is a man' appear to be identical in form; in both, the subject appears as definite and singular, and, in both, the notion of a class to which these singular subjects are referred appears to be involved. Our immediate purpose is to admit the analogy, but to emphasise the differences between these two kinds of propositions, in which common logic would have said we refer a certain object to a class.

I propose to call such terms as colour and shape *determinables* in relation to such terms as red and circular which will be called *determinates*; and, in introducing this new terminology, to examine the distinction between the relation of *red* to *colour* and the relation of *Plato* to *man*. To predicate *colour* or *shape* of an object obviously characterises it less determinately than to predicate of it *red* or *circular*; hence the former adjectives may be said negatively to be indeterminate compared with the latter. But, to supplement this negative account of the determinable, we may point out that any one determinable such as colour is distinctly other than such a determinable as shape or tone; i.e. colour is not adequately described as indeterminate, since it is, metaphorically speaking, that from which the specific determinates, red, yellow, green, etc., emanate; while

from shape emanate another completely different series of determinates such as triangular, square, octagonal, etc. Thus our idea of this or that determinable has a distinctly positive content which would be quite inadequately represented by the word 'indeterminate.' Further, what have been assumed to be determinables—e.g. colour, pitch, etc.—are ultimately *different*, in the important sense that they cannot be subsumed under some one higher determinable, with the result that they are incomparable with one another; while it is the essential nature of determinates under any one determinable to be comparable with one another. The familiar phrase 'incomparable' is thus synonymous with 'belonging to different determinables,' and 'comparable' with 'belonging to the same determinable'; not that this is the actual meaning of the terms, but that enquiry into the reason for the comparability or incomparability of two qualities will elicit the fact that they belong to the same or to different determinables respectively. This phrase 'belonging to' is also more usually used of a member of a class in relation to its class: we have, then, to contrast the significance of the relation 'belonging to' when applied in one case to a determinate and its determinable, and in the other to an individual and its class. If it is asked why a number of different individuals are said to belong to the same class, the answer is that all these different individuals are characterised by some the same adjective or combination of adjectives. But can the same reason be given for grouping red, yellow and green (say) in one class under the name colour? What is most prominently notable about red, green and yellow is that they are different, and even, as we may say, opponent

to one another; is there any (secondary) adjective which analysis would reveal as characterising all these different (primary) adjectives? In my view there is no such (secondary) adjective; in fact, the several colours are put into the same group and given the same name colour, *not* on the ground of any partial agreement, but on the ground of the special kind of difference which distinguishes one colour from another; whereas no such difference exists between a colour and a shape. Thus red and circular are adjectives between which there is no relation except that of non-identity or otherness; whereas red and blue, besides being related as non-identical, have a relation which can be properly called a relation of difference, where difference means more than mere otherness. (What is here true of colour is true of shape, pitch, feeling-tone, pressure, and so on: the ground for grouping determinates under one and the same determinable is not any partial agreement between them that could be revealed by analysis, but the unique and peculiar kind of difference that subsists between the several determinates under the same determinable, and which does not subsist between any one of them and an adjective under some other determinable.) If this is granted, the relations asserted in the two propositions 'Red is a colour' and 'Plato is a man,' though *formally* equivalent, must yet be contrasted on the ground that the latter but not the former is based upon an adjectival predication. For the latter is equivalent to predicating the adjective 'human' of 'Plato,' while, without denying that some adjectives may properly be predicated of (the adjective) red, yet the proposition 'Red is a colour' is *not* equivalent to predicating any adjective of red.

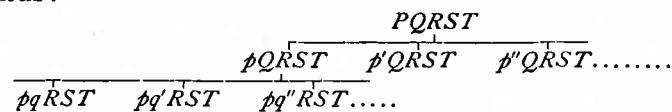
§ 2. Bearing in mind this distinction, the question arises whether what are called abstract names can be divided in the same way as concrete names into singular and general; in other words, whether adjectives can be divided into these two classes. The answer seems to be that adjectives can be divided into two classes more or less analogous to the singular and general which distinguish substantives, but that the two different kinds of adjectives are preferably distinguished as determinate and indeterminate. When, in considering different degrees of determinateness, the predication of one adjective is found to imply another, but not conversely, then the former we shall call a super-determinate of the latter and the latter a sub-determinate of the former. Thus the relation of super-determinate to sub-determinate means not only that the former is more determinate than the latter, but also that the predication of the former would imply that of the latter. A simple example can be taken from the determinable 'number': thus 7 is super-determinate to 'greater than 3'; the adjective 'greater than 3,' though not itself a *summum* determinable, may be called determinable, inasmuch as it is not merely indeterminate but capable of being further determined in the sense that it generates a definite series of determinates. To illustrate more precisely what is meant by 'generates'; let us take the determinable 'less than 4'; then 'less than 4' generates '3' and '2' and '1' in the sense that the understanding of the meaning of the former carries with it the notion of the latter. Now no substantive class-name generates its members in this way; take, for instance, 'the apostles of Jesus,' the understanding of this class-name carries with it the notion

'men summoned by Jesus to follow him,' but it does not generate 'Peter and John and James and Matthew etc.,' and this fact constitutes one important difference between the relation of sub-determinate to super-determinate adjectives and that of general to singular substantives.

§ 3. Another equally significant difference is brought out by considering that aspect of substantive-classes in which—to use the terminology of formal logic—increase of intension is accompanied by decrease of extension. The phrase 'increase of intension' conjures up the notion of adding on one attribute after another, by the logical process called conjunction; so that, taking  $p, q, r$ , to be three adjectives, increase in intension would be illustrated by regarding  $p, q, r$  conjoined as giving a greater intension than  $p, q$ ; and  $p, q$  as giving greater intension than  $p$ . We have now to point out that the increased determination of adjectival predication which leads to a narrowing of extension may consist—not in a process of conjunction of separate adjectives—but in the process of passing from a comparatively indeterminate adjective to a comparatively more determinate adjective under the same determinable. Thus there is a genuine difference between that process of increased determination which conjunctively introduces foreign adjectives, and that other process by which without increasing, so to speak, the number of adjectives, we define them more determinately.

In fact, the foreign adjective which appears to be added on in the conjunctive process, is really not introduced from outside, but is itself a determinate under another determinable, present from the start, though

suppressed in the explicit connotation of the genus. We propose to use a capital letter to stand for a determinable, and the corresponding small letter with various dashes to stand for its determinates. Thus, in passing from the genus  $p$  to the species  $pq$ , we are really passing from  $pQ$  to  $pq$ ; or again the apparent increase of intension from  $p$  to  $pq$  to  $pqr$  is more correctly symbolised as a passing from  $pQR$  to  $pqR$  to  $pqr$ . In the successive process of dividing a summum genus into the next subordinate sub-genera, and this again into sub-sub-genera, the summum genus ought to be represented by a conjunction of determinables, say  $PQRST$ ; the genera next subordinate to this, by  $pQRST, p'QRST, p''QRST$ , etc., and the genera next subordinate to the first of these by  $pqRST, pq'RST, pq''RST$ , and so on down to the *infima species* represented by determinates. Thus:



In this way we represent from the outset the nature of the ultimate individuals under the summum genus, as being characterisable jointly by the determinables  $PQRST$ , while any genus or species is represented by these same determinables, one or more of which are replaced by determinates. This meets a criticism which has often been directed against the formal account of the inverse variation of extension and intension, since we see now that the same *number* of adjectives should be used in giving the connotation of the wider as of the narrower class. To illustrate these symbols from a botanical classification of plants: let the determinable

$P$  stand for the number of cotyledons,  $Q$  for the disposition of the stamens,  $R$  for the form of the corolla,  $S$  for the attachment of the petals and sepals, and  $T$  for the divisibility of the calyx. Then  $PQRST$  represents the summum genus '*plants*' as describable under these five heads, but otherwise undetermined in character. Then  $p, p', p''$ , might stand respectively for having no cotyledons, having 1, and having 2, thus representing the defining characteristic of each of the three classes—acotyledon, monocotyledon, and dicotyledon—by the symbols  $pQRST, p'QRST, p''QRST$ . Again  $q, q', q''$ , might stand respectively for the stamens being under, around or upon the carpels, thus representing the three sub-divisions—hypogynous, perigynous, epigynous—of dicotyledons, as  $p''qRST, p''q'RST, p''q''RST$ . Taking regular and irregular to be the two possible forms of corolla, then the next sub-division under  $p''q'RST$  will be  $p''q'rST$  and  $p''q'r'ST$ . Again  $s$  and  $s'$  may stand respectively for separability and inseparability of the calyx and corolla, and yield the further sub-divisions, say  $p''q'rST, p''q'r'ST$ . The calyx may contain only one part or 3 or 4 or 5 or 6, and if these are represented respectively by  $t, t', t'', t''', t''''$ , a relatively determinate characterisation is finally symbolised by  $p''q'r's't''''$  say.

There are cases for which a modification of this general scheme is required. The cases are those in which one particular sub-division is definable by the *absence* of an element upon which the predication of other determinables depend, while in the sub-divisions co-ordinate with this the element in question is present. For example, the class of plants called acotyledons might be defined by the absence of any corolla, etc., and

hence such variations as that of the form of the corolla or the disposition of the stamens, etc., are inapplicable to this particular sub-division.  $\times \vee$

§ 4. Now adjectives under the same determinable are related to one another in various ways. One relational characteristic holds in all cases; namely that, if any determinate adjective characterises a given substantive, then it is impossible that any other determinate under the same determinable should characterise the same substantive: e.g. the proposition that 'this surface is red' is incompatible with the proposition 'this (same) surface is blue.' It has been usual to modify the above statement by adding the qualification—at the same time and at the same place; this qualification applies where the substantive extends through some period of time and over some region of space, in which case the existent substantive, having temporal or spatial parts, may be said to be extended. For this reason the qualification would perhaps better be attached to the substantive itself, and we should say that, where opponent adjectives are predicated, reference is made to different substantives, since any one part of an extended substantive is existentially *other* than any other part.

A second characteristic of many determinates under the same determinable is that the differences between different pairs of determinates can be compared with one another; so that if  $a, b, c$ , are three determinates, there are cases in which we may say that the difference between  $a$  and  $c$  is greater than that between  $a$  and  $b$ ; e.g. the difference between red and yellow is greater than that between red and orange. In this case the several determinates are to be conceived as

necessarily assuming a certain serial order, which develops from the idea of what may be called 'adjectival betweenness.' (The term 'between' is used here in a familiar metaphorical sense derived from spatial relations, and is figuratively imaged most naturally in spatial form.) Thus if  $b$  is qualitatively between  $a$  and  $c$ , and  $c$  qualitatively between  $b$  and  $d$ , and so on, the whole series has its order directly determined by the nature of the adjectives themselves. The further distinctions amongst series as interminable or as cyclic, and again of series of more than one order of dimensions, lead to logical complexities which need not be entered into here. Suffice it to say that this characteristic, which holds of so many determinates, gives significance to another well-known rule for logical division: *divisio non faciat saltum*: one meaning of which appears to be that we contemplate not merely enumerating a set of coordinate sub-classes, but enumerating them in a certain order. The rule prescribes that the order in which the sub-classes are enumerated should correspond to the order of 'betweenness' predicable of their differentiating characteristics.

The order of betweenness which characterises the determinates just considered may be either discrete or continuous. In the case of discrete series there is one determinate that can be assigned as next after any given determinate; but, in the case of a continuous series, a determinate can always be conceived as between any two given determinates, so that there are no two determinates which can be said to be next to one another in the serial order. It follows from this account of continuity that, between any two determinates which may be said to have a finite adjectival difference, may be interpolated

an indefinite number of determinates having a finite difference, and this number becomes infinite as the differences become infinitesimal. Amongst continuous series further differences between the interminable and the cyclic, and again between those of one or more order of dimensions, hold as in discrete series.

The reference here to determinables of higher or lower dimension requires explanation. Our familiar example of colour will explain the point: a colour may vary according to its hue, brightness and saturation; so that the precise determination of a colour requires us to define three variables which are more or less independent of one another in their capacity of co-variation; but in one important sense they are not independent of one another, since they could not be manifested in separation. The determinable colour is therefore *single*, though complex, in the sense that the several constituent characters upon whose variations its variability depends are inseparable. <sup>xx</sup>

§ 5. Returning to the conception of the absolutely determinate adjective, we have to note an important distinction between absolutely determinate and comparatively indeterminate predications. The distinction may thus be formulated: If, of two substantives the same *determinate* adjective can be predicated, then *all* the adjectives and relations definable in terms of the determinable, that can be predicated of the one, could be predicated of the other. But if, of two substantives the same *indeterminate* adjective can be predicated, then only *certain* of the adjectives and relations definable in terms of the determinable, that can be predicated of the one, can be predicated of the other. To illustrate

first the case of an indeterminate predication; let us take the numerical adjective 'greater than 7'; then of any collection of which this numerical adjective could be predicated, other adjectives such as 'greater than 5' and 'greater than 3' could also be predicated; but some collections that are 'greater than 7' such as the apostles, are greater than 11 and divisible by 4 for instance, whereas other collections that are 'greater than 7,' such as the muses, are less than 11 and are not divisible by 4: hence it is only *some* of the numerical adjectives that are predicable of the muses that are also predicable of the apostles, although the adjective 'greater than 7' is predicable of them both. Turning now to the case of *determinate* predication; if, instead of defining a collection by the indeterminate adjective 'greater than 7,' we had defined it by the determinate adjective 'twelve,' then any numerical adjective that is predicable of one collection of twelve, say the apostles, would be predicable of any other collection of twelve, say the months of the year or the sons of Israel; for example, 'greater than 11,' 'divisible by 4,' 'a factor of 96.' What we have here seen to hold of determinate and indeterminate number holds of any other determinable. The case of colour lends itself easily for illustration on account of the specific names which have been assigned to its determinates: thus, if the colours of two different objects are the same shade of yellow, then though these two objects may differ in any number of other respects such as shape and size, yet we may say that any colour-property of the one object will agree with the colour-property of the other; if the colour of one is more brilliant or less saturated than the colour of an orange,

then the same will hold for the colour of the other. In fact, whatever sensational determinable we take, whether it be colour, or sound, or smell, the determinate characterisations under any such determinable would lead to the same forms of generalisation that have been developed by science only in the sphere of quantity. It is agreed that in the sphere of sense perception, differences of quality strictly speaking hold only of the mental or sensational, and that the physical can only be defined in quantitative terms. Thus in the Weber-Fechner experiments the experient judges of equivalence or difference in the intensity or quality of his sensations, with which are correlated quantitative differences in the stimuli. The attempts that psychologists have made to discover formulae of correlation between the stimuli on the one hand and the sensations on the other hand show that determinateness in a qualitative or intensive scale carries with it the same logical consequences as does determinateness of magnitude for physically measurable quantities. Furthermore determinateness in either case is only approximately attainable, whether we rely upon the immediate judgments of perception or are able to utilize instruments of measurement. The practical impossibility of literally determinate characterisation must be contrasted with the universally adopted postulate that the characters of things which we can only characterise more or less indeterminately, are, in actual fact, absolutely determinate<sup>1</sup>.

<sup>1</sup> The notion of the Determinable will be shown in later chapters to have importance in a large number of applications.